Waves and Optics, Midterm, 09/12/2019

6 questions, 40 points total

Write your name and student number on each answer sheet. Use of a calculator is allowed.

Maxwell's equations and the wave equation

1. (8 points) The wave equation that follows from Maxwell's equations can be written as follows:

$$\nabla^2 \mathbf{E} - \mu_0 \epsilon_0 \frac{\partial^2 \mathbf{E}}{\partial t^2} = \mu_0 \frac{\partial \mathbf{J}_{\text{free}}}{\partial t} + \mu_0 \frac{\partial^2 \mathbf{P}}{\partial t^2} - \frac{1}{\epsilon_0} \nabla (\nabla \cdot \mathbf{P})$$

   (a) (6 points) Explain what the three terms on the right hand side represent. In doing so, explain what the symbols represent, and make sure to give examples of a material where the corresponding term is nonzero.

   (b) (2 points) Give the simplified form of the wave equation for light propagating in vacuum.

Answer model:
a) The first term represents free charges ($\mathbf{J}_{\text{free}}$), which have to be taken into account when dealing with the interaction of light with metals. The second term describe dipole oscillations ($\mathbf{P}$ is the polarization, which has a non-zero time-variation here), which play a prominent role when light propagates in nonconducting (or dielectric) materials. The third term represents anisotropic media, like crystals. (per term: +1 point for correct explanation, +1 point for example).
b) The right hand side of the equation is zero (2 points).

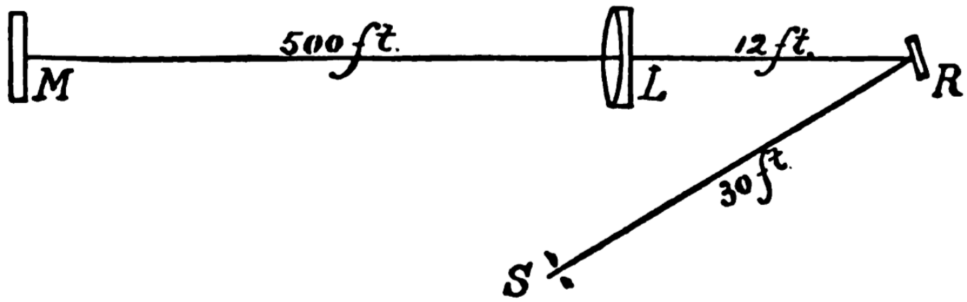
2. (8 points) The speed of light can be experimentally determined using a setup with a rotating mirror. A laser beam can be reflected of this rotating mirror. If that light beam, after having travelled down a long hallway, is reflected back onto the reflecting mirror, and then further travels back onto the front plate of the light source, it will be displaced a bit, provided that the rotation speed of the mirror is sufficiently high.

   (a) (2 points) Give a sketch of the layout of such an experiment.

   (b) (6 points) With a 1 mm laser beam diameter, and some reasonable assumptions for the distance between the source, the rotating mirror and the mirror at the end of the hallway, find the rotation frequency (radians / sec) of the mirror that is needed to make a good measurement (a displacement of 10 times the beam diameter) of the lightspeed. Check your answer. Do you think that value for the rotation speed can be obtained in practice?

Answer model:
(a) (see figure below)

Fig. 1.



(b) For a good measurement we would like to see a displacement of the laser beam of at least 10 times the diameter. If we have a light beam diameter of 1 mm, we are looking for a displacement of 1 cm. For the distance between the rotating mirror and the steady mirror we take 30 m, and for the distance between the light source and the rotating mirror we take 4 m. This means that the mirror should have rotated $\approx 0.5/400 = 1/800$ *of* 2π *radians (small angle approximation, and rotation of* θ *gives* 2θ *deflection), in the time that it takes the light to travel from the rotating mirror to the fixed mirror and back. This time is* $(2 \times 30 \text{ m})/(3 \times 10^8 \text{ m/s})$ *is* 2×10^{-7}s*, which corresponds to* $0.2\ \mu s \times 800 \times 2\pi \approx 1$ ms *for a full rotation. This is a rotation frequency of* ≈ 1 kHz*. That is pretty high, but possible (video linked to in book demonstrates* ~ 500 *Hz.)*
8 points total; 2 for sketch, 3 for reasonable assumptions, 3 for correct calculation.

Plane waves and refractive index

3. (6 points) Consider the complex index of refraction $\mathcal{N} = n + i\kappa = \sqrt{1+\chi}$.
   (a) (3 points) How are the phase-velocity, the frequency and the wavelength of the wave modified, when it travels in a material with this index of refraction?
   (b) (3 points) Explain which part of the Lorentz model, that models the susceptibility, represents the complex part of the index of refraction.

Answer model:
(a) $v_{\text{phase}} = c/n$, $\lambda = \lambda_{vac}/n$, $\omega = \omega_{\text{vac}}$. *(1 point for each).*
(b) In the Lorentz model the electrons in the material are modelled as connected with a linear restoring force to the atom (like a spring). A friction term, proportional to the velocity of the electron, represents a damping mechanism - and this is the term that corresponds to the complex part of the index of refraction. It results in damping of the electromagnetic wave, and a phase delay of the oscillation of the electron with respect to the oscillation of the electromagnetic wave. (+1 point for conceptualizing the Lorentz model, +1 point for naming friction/damping, +1 point for linking to absorption, -1/2 point for not naming phase delay)

Reflection and refraction

4. (6 points) The Fresnel coefficients represent the ratio of the reflected and transmitted field components to the incident field components, when a wave travels from a material with index of refraction n_1 to a material with index of refraction n_2.

(a) (3 points) Use your knowledge of the pattern of emission of an oscillating dipole to explain the minimum of reflected light for p-polarized light at Brewster's angle. Why does this not happen for s-polarized light?

(b) (3 points) The reflectance R is equal to the absolute square of the reflectivity coefficients, but the transmittance T is not equal to the square of the tranmission coefficients. Explain why this is not so.

Answer model:
(a) At Brewster's angle, the k-vector in the higher-density medium is oriented such that it is exactly perpendicular to the k-vector of the reflected light. The electric field for a p-polarized wave therefore oscillates exactly in the direction of the reflected wave. An oscillating dipole has zero intensity along the direction of oscillation, and therefore there can not be any reflected intensity at this angle.
(b) The transmitted intensity also depends on the ratio of the index of refraction. This can be nicely explained with geomtric arguments, as shown in figure 3.5 in the book.

Multiple parallel interfaces

5. (6 points) The fraction of power that transmits through a double-interface arrangement is given by

$$T^{\mathrm{tot}} = \frac{T^{\mathrm{max}}}{1 + F\sin^2(\frac{\Phi}{2})},$$

which is valid for both s or p polarized light. In the case of identical boundaries, the following holds:

$$T^{\mathrm{max}} = \frac{T^2}{(1-R)^2}$$

and

$$F = \frac{4R}{(1-R)^2}.$$

The parameter Φ is given by

$$\Phi = \frac{4\pi n_1 d}{\lambda_{\mathrm{vac}}}\cos(\theta_I) + 2\phi_r.$$

(a) (2 points) Make a sketch of the total transmitted power of a Fabry-Perot interferometer as a function of the phase. Do this for three different values of F, and indicate which values you have chosen, and to which curve they correspond.

(b) (4 points) With reasonable assumptions for the involved parameters, give an estimate of the sensitivity to wavelength changes for a Fabry-Perot interferometer with a finesse of 100.

Answer model:
(a) Figure 4.9 in the book.
(b) (page 102/103 in the book)

$$\Delta\lambda_{\mathrm{FSR}} = \frac{\lambda_{\mathrm{vac}}^2}{2n_1 d_0\cos\theta_1} = \Delta\lambda_{\mathrm{FSR}} = \frac{(500\mathrm{nm})^2}{2(1)(1\mathrm{cm})\cos 0\deg} = 0.0125\mathrm{nm}$$

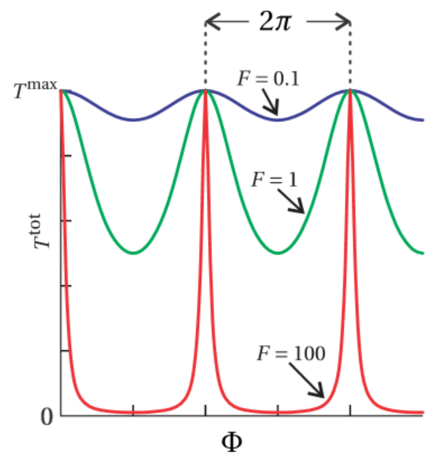


Figure 4.9 Transmittance as the phase Φ is varied. The different curves correspond to different values of the finesse coefficient.

Coherence theory

6. (6 points) Consider a Michelson interferometer.
   (a) (2 points) Make a sketch of the layout of such a device.
   (b) (2 points) Sketch the detected intensity as function of the relative distance between the optical path length of the two arms, for a monochromatic CW laser beam. Make sure to also indicate the units on the vertical axis, in term of the input intensity, I_0.
   (c) (2 points) Explain the concepts of coherence time, coherence length and spatial coherence.

Answer model:
(a) Figure 8.1 in the book. (+1 point for correct mirrors, +1 point for correct propagation of light)

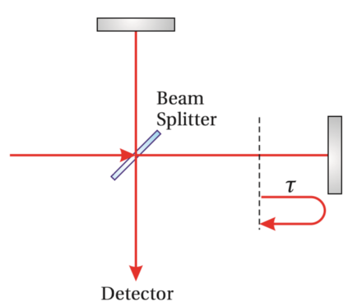


Figure 8.1 Michelson interferometer.

(b) Figure 8.2 in the book. The vertical axis should have $4I_0$ *as maximum value. (+1 point for*

shape of wave, +1 point for axes/scaling)

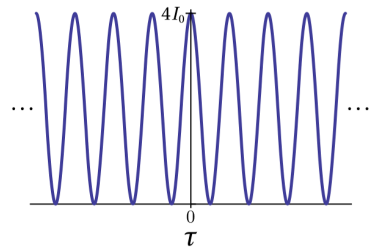


Figure 8.2 The intensity seen at the detector of a Michelson interferometer with a plane-wave

(c) Use of a pinhole to limit the spatial extent of the source; use of a compensating glass plate; make sure that the optical path length is exactly the same in both arms. (+1 point for correct coherence time/length, +1 point for spatial coherence)